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Formula Sheet

**Formula Sheet Sections 3.6 – 5**

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# Definition 3.9

A random variable Y is said to have a negative binomial probability distribution

if and only if

# Theorem 3.9

If Y is a random variable with a negative binomial distribution,

# Definition 3.10

A random variable Y is said to have a hypergeometric probability distribution

if and only if

where y is an integer 0, 1, 2, . . . , n, subject to the restrictions y ≤ r and

n− y ≤ N− r .

# Theorem 3.10

If Y is a random variable with a hypergeometric distribution,

# Definition 3.11

A random variable Y is said to have a Poisson probability distribution if and

only if

# Theorem 3.11

If Y is a random variable possessing a Poisson distribution with parameter λ,

then

# Definition 3.12

The kth moment of a random variable Y taken about the origin is defined to be

E( ) and is denoted by .

# Definition 3.13

The kth moment of a random variable Y taken about its mean, or the kth central

moment of Y, is defined to be E[(] and is denoted by μk.

# Definition 3.14

The moment-generating function m(t) for a random variable Y is defined to be

m(t)= E(). We say that a moment-generating function for Y exists if there

exists a positive constant b such that m(t) is finite for |t| ≤ b.

# Theorem 3.12

If m(t) exists, then for any positive integer k,

In other words, if you find the kth derivative of m(t) with respect to t and

then set t = 0, the result will be

# Definition 3.15

Let Y be an integer-valued random variable for which P(Y= i )= , where

i= 0, 1, 2, . . . . The probability-generating function P(t) for Y is defined to

be

for all values of t such that P(t) is finite.

# Definition 3.16

The kth factorial moment for a random variable Y is defined to be

where k is a positive integer.

# Theorem 3.13

If P(t) is the probability-generating function for an integer-valued random

variable, Y , then the kth factorial moment of Y is given by

# Theorem 3.14

**Tchebysheff’s Theorem** Let Y be a random variable with mean μ and finite

variance σ 2. Then, for any constant k > 0,

# Definition 4.1

Let Y denote any random variable. The distribution function of Y , denoted by

F(y), is such that F(y)= P(Y ≤ y) for −∞ < y < ∞.

# Theorem 4.1

Properties of a Distribution Function1 If F(y) is a distribution function, then

1. F(−∞) ≡

2. F(∞) ≡

3. F(y) is a nondecreasing function of y. [If and are any values such

that < , then F() ≤ F().]

# Definition 4.2

A random variable Y with distribution function F(y) is said to be continuous

if F(y) is continuous, for −∞ < y <

# Definition 4.3

Let F(y) be the distribution function for a continuous random variable Y . Then

f (y), given by

wherever the derivative exists, is called the probability density function for the

random variable Y.

# Theorem 4.2

**Properties of a Density Function** If f (y) is a density function for a continuous

random variable, then

1. F(y) ≥ 0 for all y; −∞ < y < ∞.

# Definition 4.4

Let Y denote any random variable. If 0 < p < 1, the pth quantile of Y,

denoted by , is the smallest value such that P(Y ≤ ) = F( ) ≥ p. If Y

is continuous, is the smallest value such that F( ) = P(Y ≤ ) = p.

Some prefer to call the 100pth percentile of Y.

# Theorem 4.3

If the random variable Y has density function f (y) and a < b, then the proba-

bility that Y falls in the interval [a, b] is

# Definition 4.5

The expected value of a continuous random variable Y is

provided that the integral exists.

# Definition 4.4

Let g(Y ) be a function of Y ; then the expected value of g(Y ) is given by

provided that the integral exists.

# Theorem 4.5

Let c be a constant and let g(Y ), (Y ), ( (Y ), . . . , (Y ) be functions of a

continuous random variable Y. Then the following results hold:

1. E(c) = c.
2. E[cg(Y)] = cE[g(Y)].
3. E[(Y)+(Y)+ *…*+(Y)] = E[(Y)]+ E[(Y)]+ *…*+(Y)]

# Definition 4.6

If < , a random variable Y is said to have a continuous uniform probability

distribution on the interval (, ) if and only if the density function of Y is

# Definition 4.7

The constants that determine the specific form of a density function are called

parameters of the density function.

# Theorem 4.6

If and Y is a random variable uniformly distributed on the interval (

# Definition 4.8

A random variable Y is said to have a normal probability distribution if and

only if, for σ > 0 and −∞ < μ < ∞, the density function of Y is

# Theorem 4.7

If Y is a normally distributed random variable with parameters μ and σ , then

E(Y) =

# Definition 4.9

A random variable Y is said to have a gamma distribution with parameters

α > 0 and β > 0 if and only if the density function of Y is

Where

# Theorem 4.8

If Y has a gamma distribution with parameters α and β, then

# Definition 4.10

Let ν be a positive integer. A random variable Y is said to have a chi-square

distribution with ν degrees of freedom if and only if Y is a gamma-distributed

random variable with parameters α = and β= 2.

# Theorem 4.9

If Y is a chi-square random variable with ν degrees of freedom, then

μ = E(Y )= ν and = V (Y )= 2ν.

# Definition 4.11

A random variable Y is said to have an exponential distribution with parameter

β > 0 if and only if the density function of Y is

# Theorem 4.10

If Y is an exponential random variable with parameter β, then

# Definition 4.13

If Y is a continuous random variable, then the kth moment about the origin is

given by

The kth moment about the mean, or the kth central moment, is given by

# Definition 4.14

If Y is a continuous random variable, then the moment-generating function of

Y is given by

The moment-generating function is said to exist if there exists a constant b > 0

such that m(t) is finite for |t| ≤ b.

# Theorem 4.12

Let Y be a random variable with density function f (y) and g(Y ) be a function

of Y . Then the moment-generating function for g(Y ) is

# Theorem 4.13

**Tchebysheff’s Theorem** Let Y be a random variable with mean μ and finite

variance σ 2. Then, for any constant k > 0,

# Definition 4.15

Let Y have the mixed distribution function

and suppose that is a discrete random variable with distribution function

(y) and that is a continuous random variable with distribution function

(y). Let g(Y ) denote a function of Y . Then

E[g(Y)] = .

# Definition 5.1

Let and be discrete random variables. The joint (or bivariate) probability

function for and is given by

# Theorem 5.1

If are discrete random variables with joint probability function

p(y1, y2), then

1. where the sum is over all values ( that are assigned nonzero probabilities.

# Definition 5.2

For any random variables and , the joint (bivariate) distribution function

F is

# Definition 5.3

Let and be continuous random variables with joint distribution function F(). If there exists a nonnegative function f(, such that

F( = ,

for all −∞ < < ∞, −∞ < < ∞, then and are said to be jointly

continuous random variables. The function f (, ) is called the joint prob-

ability density function.

# Theorem 5.2

If and are random variables with joint distribution function F(, ), then

1. F(−∞, −∞) = F(−∞, ) = F(, −∞) = 0.
2. F(∞, ∞)= 1.
3. If

F() – F() – F(,+ F() ≥ 0.

# Definition 5.4

a Let and be jointly discrete random variables with probability function

p(, ). Then the marginal probability functions of and , respectively,

are given by

and

b Let and be jointly discrete random variables with probability function

f(, ). Then the marginal probability functions of and , respectively,

are given by

and

# Definition 5.5

If and are jointly discrete random variables with joint probability function

p(, ) and marginal probability functions (and (, respectively,

then the conditional discrete probability function of given is

p(

provided that

# Definition 5.6

If and are jointly continuous random variables with joint density function

f(, ), then the conditional distribution function of given = is

= P(|

# Definition 5.7

Let and be jointly continuous random variables with joint density f(, ), and marginal densities ( and ), respectively. For any such that ) > 0,

the conditional density of , given = is given by

and for any such that > 0, the conditional density of given = is given by

# Definition 5.8

Let have distribution function Let , have distribution function ,

and and have joint distribution function F(, *.* Then and are said to be independent if and only if

F(,=

for every pair of real numbers ().

If and are not independent, they are said to be dependent.

# Theorem 5.4

If and are discrete random variables with joint probability function p( and marginal probability function and , respectively, then and are independent if and only if

p(,=

for all pairs of real numbers (, *.*

If and are continuous random variables with joint density function f ()

and marginal density functions () and (), respectively, then and

are independent if and only if

f(,=

for all pairs of real numbers (, *.*

# Theorem 5.5

Let and have a join density f(, that is positive if and only if a ≤ ≤ b and c ≤ ≤ d, for constants a, b, c, and d: and f(, = 0 otherwise. Then and are independent random variables if and only if

f(,=

where g( is a nonnegative function of alone and h( is a nonnegative function of alone.

# Theorem 5.6

Let c be a constant. Then

E(c) = c.

# Theorem 5.7

Let g() be a function of the random variable and and let c be a constant. Then

E[cg()] = cE[g()].

# Theorem 5.8

Let and be random variables and (), (),….,() be functions of and Then

E[() + () + … + ()]

= E[()] + ()] + … +E[()].

# Theorem 5.9

Let and be independent random variables and g() and h() be functions of only and , respectively. Then

E[g()h()] = E[g()]E[h()],

Provided that the expectations exist.

# Definition 5.10

If and are random variables with means and , respectively, the covariance of and is

Cov() = E[()( - )].

# Theorem 5.10

If and are random variables with means and , respectively, then

Cov() = E[()( - )] = E() – E(E(.

# Theorem 5.11

If and are independent random variables, then

Cov(,) = 0.

Thus, independent random variables must be uncorrelated.

# Definition 5.13

If and are any two random variables, the conditional expectation of g(), given that = , is defined to be

E(g() | = ) =

If and are jointly continuous and

E(g() | = ) =

If and are jointly discrete.

# Theorem 5.14

Let and denote random variables. Then

E( = E[E(|)].

where on the right-hand side the inside expectation is with respect to the con-

ditional distribution of given and the outside expectation is with respect

to the distribution of .

# Theorem 5.15

Let and denote random variables. Then

V( = E[V( | )] + V[E( | )].